Pi and Golden Number: not random occurrences of the ten digits.

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Abstract: This paper demonstrates that the order of first appearance of the ten digits of the decimal system in the two most fundamental mathematical constants such as the number Pi and the Golden Number is not random but part of a arithmetical logic. This arithmetical logic is identical to Pi to its inverse and to the Golden Number. The same arithmetical phenomenon also operates in many other constants whose square roots of numbers 2, 3 and 5, the first three prime numbers.

1. Introduction.

The number Pi (π) and the Golden Number (φ) and the inverse of these numbers are made up of a seemingly random digits. This article is about order of the first appearance of the ten figures of decimal system in these fundamental numbers of mathematics. There turns out that the ten digits decimal system (combined here with their respective numbers: figure 1 = number 1, figure 2 = number 2, etc..) do not appear randomly in the digits sequence of Pi (π) and the digits sequence of Golden Number (φ). The same phenomenon is also observed for the inverse of these two numbers (1/π et 1/φ).

1.1. Method.

This article studies the order of the first appearance of the ten figures of the decimal system in the decimals of constants (or numbers). After location of these ten digits merged then in numbers (figure 1 = number 1, etc), an arithmetical study of these is introduced.

Constant	Number and its n first decimals*	Order of appearance of figures
π	3.**14**1**5926**5**3**5**8**9**7**9323846264338327950**2**...	1 4 5 9 2 6 3 8 7 0

Fig. 1. Analytical process of constants. * n first sufficient decimals for study.

2. The ratio 3/2.

The sum of the ten figures of the decimal system, **considered as numbers in this article,** is 45:

$$0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$$

This number is sum of two others: 45 = 27 + 18. These two numbers have a ratio to 3/2 and are respectively equal for 3 times and twice 9. The number 10, which here represents the ten possible occurrence ranks of the ten figures of the decimal system, has the same characteristics: sum of two other one numbers with a ratio to 3/2: 10 = 6 + 4.

2.1. The ratio 3/2 inside constants π and φ.

Figure 2 analyses the constant Pi (π). In this table, the ten digits of the decimal system are identified (a) and ranked in order of their first appearance (c). At last arithmetical analysis is presented: the sum of the first six values and the last four in a ratio to 3/2. All tables in this article use the same type of set-up with an arithmetical area (d) more or less developed.

a	π = 3.**14**1**5926**5**3**5**8**9**7**93238462643383279502**0**2...									
b	1	2	3	4	5	6	7	8	9	10
c	1	4	5	9	2	6	3	8	7	0
d	**27 (3 × 9)**						**18 (2 × 9)**			

Fig. 2. Suite of appearance of digits in π. **a**: constant and location of the appearances of the 10 figures of the decimal system. **b**: rank of appearance order (from 1 to 10). **c**: digits classified in order of appearance. **d**: arithmetical grouping.

There appears that for Pi, the ten digits of the decimal system are organized in a ratio to 3/2: the sum of first six digits is to 27 and the last four to 18. This configuration has a probability of occurrence [1] to 1/11.66. Thus, 91.43% of possible combinations of onset did not this ratio. Figure 3 analyses the constant 1/Pi (1/π). The same phenomenon is observed for this constant. The probability [2] that such a phenomenon occur simultaneously for a constant and its inverse is to 1/23.33. Only the constant Phi (φ), by its arithmetical nature, has of course this property.

1/π = 0.**318**3**09**88**6**183**7**90671**5**377675**2**67**4**503…									
1	2	3	4	5	6	7	8	9	10
3	1	8	0	9	6	7	5	2	4
27 (3 × 9)						**18 (2 × 9)**			

Fig. 3. Analysis of the constant 1/π.

The same phenomenon (Fig. 4) of ratio to 3/2 (27/18) is present in the constant Phi (φ) and of course in 1/φ.

φ * = 1.**61803**3**9**88**74**9894848**2**04**5**868…									
1	2	3	4	5	6	7	8	9	10
6	1	8	0	3	9	7	4	2	5
27 (3 × 9)						**18 (2 × 9)**			

Fig. 4. Analysis of the constant Phi. * By its very nature, φ and its inverse have the same decimals. These two numbers are therefore confused in this study.

Also, there is determined (Fig. 5) that the ten digits of constants 1/π and 1/φ split identically in both fractions to ratio 3/2: the same first six and last four digits.

Constant	Order of appearance of the 10 digits	Sharing out digits	
		first 6 digits	last 4 digits
1 /π	3 1 8 0 9 6 7 5 2 4	3 1 8 0 9 6	7 5 2 4
1/φ (or φ)	6 1 8 0 3 9 7 4 2 5	6 1 8 0 3 9	7 4 2 5

Fig. 5. Similarity of appearance of digits inside 1/π and 1/φ.

This double shape has only one likelihood of appearance [3] to 1/210. So, 99.52 % of combinations of appearance of figures do not have this shape.

2.2. The ratio 3/2 inside other constants.

This phenomenon of ratio to 3/2 (27/18) is present in other significant constants. This arithmetical phenomenon is not therefore haphazard. This phenomenon is present in constants $\sqrt{5}$, ζ (5) (Zeta 5 function), number e (constant of Neper), in constants of Copeland and Kaprekar. Also, in significant fractions relating directly to the decimal system as the fraction 9876543210/0123456789.

Constants	Location of appearance of digits*	Sharing out 10 digits (6 and 4 classified digits)	
$\sqrt{5}$	2.**2360**6**79**77**4**997**8**9696409**1**736 ….**5**…	2 3 6 0 7 9	4 8 1 5
ζ (5) (Zeta 5)	1.**036927**7**5**5**14**33699263313654**8**…	0 3 6 9 2 7	5 1 4 8
1467/6174 (constant of Kaprekar)	0.**237609**3294460 6**41**399416…**5**…**8**…	2 3 7 6 0 9	4 1 5 8
9876543210/0123456789	80.**0**..00**729**0..0**6**6**3**39000603 6**84**9**1**…**5**…	0 7 2 9 6 3	8 4 1 5
e (constant of Neper)	2.**7182**81828**4590**452**3**53**6**…	7 1 8 2 4 5	9 0 3 6
Copeland constant	0.**23571**113171**9**….**4**…..**6**……**8**…..**0**…	2 3 5 7 1 9	4 6 8 0
Landau-Ramanujan constant	0.**7642**2**3**6**5**35**89**22**0**66299069873**1**…	7 6 4 2 3 5	8 9 0 1

Fig. 6. Constants with ratio to 3/2 (27/18) by the order of the first appearance of the figures of their decimals. * The dotted replace too much of numbers insignificant (already occurred).

Constants	Location of appearance of digits*	Sharing out 10 digits (6 and 4 classified digits)	
9/12345	0.0…729040097205346...17253948…	0 7 2 9 4 5	3 6 1 8
12345/67890	0.18183826778612461334511710119…	1 8 3 2 6 7	4 5 0 9
12345/56789	0.21738364824173695610…	2 1 7 3 8 6	4 9 5 0
13579/97531	0.13922752765787288144282330…	1 3 9 2 7 5	6 8 4 0
543212345/123454321	4.400107996219913598…	4 0 1 7 9 6	2 3 5 8
235711/117532 (5 n prime numbers)	2.005504883776333253922…044651…	0 5 4 8 3 7	6 2 9 1
3φ/2	2.427050983124842272306…	4 2 7 0 5 9	8 3 1 6
$3\varphi\sqrt{3-\varphi}$	5.7063390977709214326986 …5…	7 0 6 3 9 2	1 4 8 5
(*φ* +3)/4	1.154508497187473712051146…	1 5 4 0 8 9	7 3 2 6
$4\sqrt{\pi}$ *	7.08981540362…7…	0 8 9 1 5 4	3 6 2 7
$x \Rightarrow x^3 - 2x = (2\varphi - 1)^2$	2.094551481542326…7…	0 9 4 5 1 8	2 3 6 7
log2/log3**	0.6309297535714…8…	6 3 0 9 2 7	5 1 4 8

Fig. 7. Other constants with ratio to 3/2 (27/18) by the order of the first appearance of the figures of their decimals. *4 $\sqrt{\pi}$ = perimeter of a square having as surface = π. ** log2/log3 = fractal dimension of the Cantor set.

One note that, as for constants 1/π and 1/φ, the ten digits of the constants grouped in Figure 8 are distributed identically in the two fractions of the ratio 3/2 with the same first six and last four digits, although there are 210 possibilities [3] for the division into six and four figures in order of appearance of digits in their decimals suite.

constant	Order of appearance of the 10 digits	Sharing out 10 digits (6 and 4 classified digits)	
$\sqrt{5}$	2 3 6 0 7 9 4 8 1 5	2 3 6 0 7 9	4 8 1 5
ζ (5) (Zeta 5)	0 3 6 9 2 7 5 1 4 8	0 3 6 9 2 7	5 1 4 8
1467/6174 (constante de Kaprekar)	2 3 7 6 0 9 4 1 5 8	2 3 7 6 0 9	4 1 5 8
$3\varphi\sqrt{3-\varphi}$	7 0 6 3 9 2 1 4 8 5	7 0 6 3 9 2	1 4 8 5
9876543210/0123456789	0 7 2 9 6 3 8 4 1 5	0 7 2 9 6 3	8 4 1 5
log2/log3	6 3 0 9 2 7 5 1 4 8	6 3 0 9 2 7	5 1 4 8

Fig. 8. Similarity of appearance of digits in these 6 constants: the same first six and last four digits.

It will be shown later (Chapter 5.3) that this combination of six and four digits is not random but occurs by much greater propensity than is possible in according to probabilities.

3. Areas by 1, 2, 3 and 4 figures in the fundamental constants.

π, 1/π, 1/φ and other constants (see 3.1) share another peculiar arithmetical property. Alongside the phenomenon of ratio to 3/2, their digits are divided to form four areas of occurrence which are always by sums of multiples of number 9:

Constant	π = 3.14159265358979323846264338327950 2…									
Rank	1	2	3	4	5	6	7	8	9	10
Digits occurrence	**1**	**4**	**5**	**9**	**2**	**6**	**3**	**8**	**7**	**0**
Occurrence areas	Area 3	Area 2	Area 2	Area 1	Area 3	Area 3	Area 4	Area 4	Area 4	Area 4

Fig. 9. Identification, for Pi, to 4 arithmetical areas which are by sums of multiples of number 9.

In these constants, the sums of digits of four areas of appearance (which size is regularly progressive) are always by multiples of the number 9. These zones are formed by 1, 2, 3 and 4 ranks of digits appearance. Also, these areas (see fig. 10) are always identical in according to the occurrence rank:

- area by 1 figure: rank 4
- area by 2 figures: ranks 2 - 3
- area by 3 figures: ranks 1 - 5 - 6
- area by 4 figures: ranks 7 - 8 - 9 -10

π = 3.**14**1**5926**5**3**5**8**9**7**93238462643383279**50**2…									
1	2	3	4	5	6	7	8	9	10
1	**4**	**5**	**9**	**2**	**6**	**3**	**8**	**7**	**0**
	9 (1 x 9)		**9** (1 x 9)						
		9 (1 x 9)				**18 (2 × 9)**			
		27 (3 × 9)							
1/π = 0.**318**3**09**88**6**183**7**90671**5**37767 5**2**67**4**503…									
1	2	3	4	5	6	7	8	9	10
3	**1**	**8**	**0**	**9**	**6**	**7**	**5**	**2**	**4**
	9 (1 x 9)		**0** (0 x 9)						
		18 (2 x 9)				**18 (2 × 9)**			
		27 (3 × 9)							
1/φ = 0.**61803**3**9**88**74**98948482**0**4**5**868…									
1	2	3	4	5	6	7	8	9	10
6	**1**	**8**	**0**	**3**	**9**	**7**	**4**	**2**	**5**
	9 (1 x 9)		**0** (0 x 9)						
		18 (2 x 9)				**18 (2 × 9)**			
		27 (3 × 9)							

Fig. 10. Analysis of constants π, 1/π and 1/φ with put in an obvious place by 4 identical arithmetical areas.

This number 9 is the greatest divisor of 45, the sum of the ten digits of the decimal system. The likelihood of appearance of this arithmetical arrangement [4] is only to 1/420 for every constant. 99.76 % of possible combinations do not have this configuration. It seems therefore not very unlikely that precisely, Pi, Phi and their inverses share this property.

3.1. Other constants with the same properties.

Inside constants which are presented figure 11, We note that, always with the same probability of 1/420 and as for π, 1/π, 1/φ, the ten digits are by the same four arithmetical areas so as to form four values which are multiple of 9. This with a ratio to 3/2 between the first six and last four digits which occurred:

Constants	Order of appearance of the 10 digits	Sharing out 10 digits				
		Areas by 1, 2 and 3 digits				Areas by 4 digits
$\sqrt{5}$	2 3 6 0 7 9 4 8 1 5	2	3 6	0	7 9	4 8 1 5
ζ (5)	0 3 6 9 2 7 5 1 4 8	0	3 6	9	2 7	5 1 4 8
9876543210/0123456789	0 7 2 9 6 3 8 4 1 5	0	7 2	9	6 3	8 4 1 5
9/12345	0 7 2 9 4 5 3 6 1 8	0	7 2	9	4 5	3 6 1 8
$\frac{3\varphi}{2}$	4 2 7 0 5 9 8 3 1 6	4	2 7	0	5 9	8 3 1 6
$\frac{3+\varphi}{4}$	1 5 4 0 8 9 7 3 2 6	1	5 4	0	8 9	7 3 2 6

Fig. 11. Other constants with highlighted of 4 arithmetical areas which are by multiples of 9. Probability to 1/420.

3.2. Similarity between the constants 1/π and 1/φ.

About constants 1/π, and 1/φ, it has been demonstrated that, in order of first appearance places of digits of their decimals, both have the same ratio to 3/2, also, both in this division the same first six and last four digits, both spread their digits to form the same four arithmetical areas which are multiple of 9. It is finally that, for these two fundamental constants, the same figures appear in the same four areas of 1, 2, 3 and 4 digits. The probability [5] of the occurrence of such a arithmetical phenomenon is only to 1/12600.

Occurrence ranks ⇒	1	2	3	4	5	6	7	8	9	10
$\frac{1}{\pi}$ = 0.**318**3**09**88**6**183**7**90671**5**37767**5**2**6**7**4**5	**3**	**1**	**8**	**0**	**9**	**6**	**7**	**5**	**2**	**4**
$\frac{1}{\varphi}$ = 1.**61803**3**9**88**74**98948482**0**4**5**86834365	**6**	**1**	**8**	**0**	**3**	**9**	**7**	**4**	**2**	**5**
Occurrence areas ⇒		Area 2		Area 1			Area 4			
		Area 3								

Fig. 12. Constants 1/π and 1/φ : the same figures in the 4 areas of appearance. Probability [5] to 1/12600.

So, the two most prime mathematical constants such as Pi and the Golden Number are they bound by these strange phenomena. The order of their decimal has nothing random about all that arithmetical phenomena similar to recur in other significant constants. The same phenomenon occurs (probability to 1/12600) between the constant ζ(5) (Zeta 5 function) and the number $3-\sqrt{5}$ which is the decimal complementarity of $\sqrt{5}$.

Occurrence ranks ⇒	1	2	3	4	5	6	7	8	9	10
ζ(5) = 1.**036927**7**5**5**14**336992633136548…	**0**	**3**	**6**	**9**	**2**	**7**	**5**	**1**	**4**	**8**
$3-\sqrt{5}$ = 0.**7639**3**20**22**5**0.2**1**03…90**8**26**4**3…	**7**	**6**	**3**	**9**	**2**	**0**	**5**	**1**	**8**	**4**
Occurrence areas ⇒		Area 2		Area 1			Area 4			
		Area 3								

Fig. 13. Constants ζ(5) and $3-\sqrt{5}$: the same figures in the 4 areas of appearance. Probability [5] to 1/12600.

4. Similar phenomena with other constants.

4.1. Constants $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$.

A similar phenomenon appears for constants $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ which are three fundamental constants of mathematics: the square roots of the first three prime numbers. As in π, in numbers $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$, sums of the same groups described above (4 areas of digits occurrence) have always values which are by multiples of the same number: 3 to $\sqrt{2}$, 5 to $\sqrt{3}$ and 9 to $\sqrt{5}$. These three values are the three possible divisors of 45, which is the sum of ten figures of decimal system. The probability of occurrence [6] such configurations is to 1/18 and only 5.55 % of all possible combinations (digits occurrences) have these properties.

It is remarkable that this phenomenon occurs precisely for Pi, Phi (their inverses also) and for the square roots of the first three primes (prime number after having this feature is the 103 number located in 27th position in the sequence of primes).

$\sqrt{2}$ = 1.41421356237309504880 1…									
1	2	3	4	5	6	7	8	9	10
4	**1**	**2**	**3**	**5**	**6**	**7**	**0**	**9**	**8**
	3 (1 × 3)		**3** (1 × 3)						
		15 (5 × 3)					**24 (8 × 3)**		
		21 (7 × 3)							
$\sqrt{3}$ = 1.7320508075688772935274463415 05…									
1	2	3	4	5	6	7	8	9	10
7	**3**	**2**	**0**	**5**	**8**	**6**	**9**	**4**	**1**
	5 (1 × 5)		**0** (0 × 5)						
		20 (4 × 5)					**20 (4 × 5)**		
		25 (5 × 5)							
$\sqrt{5}$ = 2.2360679774997896964091736 ….5…									
1	2	3	4	5	6	7	8	9	10
2	**3**	**6**	**0**	**7**	**9**	**4**	**8**	**1**	**5**
	9 (1 × 9)		**0** (0 × 9)						
		18 (2 × 9)					**18 (2 × 9)**		
		27(3 × 9)							

Fig. 14. Analysis of constants $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$: same arithmetical constructions.

It also notes the increasing order of the divisor for these three constants: 3 to $\sqrt{2}$, 5 to $\sqrt{3}$ et 9 to $\sqrt{5}$.

4.2. Variants of constants $\sqrt{2}$ and $\sqrt{3}$.

Two variants of the constants $\sqrt{2}$ and $\sqrt{3}$ are organized in remarkably identical configurations. Their four arithmetical areas (identical to those defined above) are multiples of the same divisor (3) and their main ratio (6/4 digits) is the same (11/4).

1/[(1/$\sqrt{2}$) + 1] = 0.585786437626904951…									
1	2	3	4	5	6	7	8	9	10
5	**8**	**7**	**6**	**4**	**3**	**2**	**9**	**0**	**1**
	15 (5 × 3)		**6** (2 × 3)						
		12 (4 × 3)					**12** (4 × 3)		
		33 (11 × 3)							
1/[(1/$\sqrt{3}$) + 2] = 0.387995381130102064…									
1	2	3	4	5	6	7	8	9	10
3	**8**	**7**	**9**	**5**	**1**	**0**	**2**	**6**	**4**
	15 (5 × 3)		**9** (3 × 3)						
		9 (3 × 3)					**12** (4 × 3)		
		33 (11 x 3)							

Fig. 15. Analysis of constants, variants of $\sqrt{2}$ and $\sqrt{3}$: same arithmetical constructions.

4.3. Constant $\sqrt{4.5}$

The sum of the ten digits of the decimal system is 45, the average of these ten figures is therefore 4.5 Remarkable phenomena appear in the constant $\sqrt{4.5}$.

This constant has the same general phenomena described in this paper: prime ratio whose two quotients (here 15/30) are multiples of a divisor of 45 and the same groups of 1, 2, 3 and 4 figures which are by multiples to the same divisor of 45 (here 5). But also, two other strange phenomena emerge.

$\sqrt{4.5}$ = 2.12132034355964257320253308631…									
1	2	3	4	5	6	7	8	9	10
1	**2**	**3**	**0**	**4**	**5**	**9**	**6**	**7**	**8**
	5 (1 × 5)		**0** (0 × 5)			**30** (6 x 5)			
10 (2 x 5)									
15 (3 x 5)									

Fig. 16. Analysis of constant $\sqrt{4.5}$

First phenomenon: the first six digits (0 to 5) of the decimal system is precisely the group of top six. The probability [3] of the occurrence of this combination is to 1/210. Second phenomenon: from the first to the tenth place, the figures are so perfectly symmetrical, forming groups of two numbers whose total is always equal to 9. The arithmetical probability [7] by which this occurs is to 1/945.

$\sqrt{4.5}$ = 2.12132034355964257320253308…										
ranks	1	2	3	4	5	6	7	8	9	10
figures	1	2	3	0	4	5	9	6	7	8
sums of values					9					
				9						
			9							
		9								
	9									

Fig. 17. Symmetrical sharing out of figures in the constant $\sqrt{4.5}$. Probability to 1/945.

4.3.1. Constants $\sqrt{4.5}$ and $((\pi\text{-}2)/\pi)^2$

In the development of its decimal digits, the constant $\sqrt{4.5}$ has a similar arrangement to the number derived from Pi, $((\pi-2)/\pi)^2$. This number is the result of equation: $1-\dfrac{1}{\pi}=\dfrac{\sqrt{x}+1}{2}$

This number is not arbitrary, this equation is similar to the equation $1+\dfrac{1}{\varphi}=\dfrac{\sqrt{x}+1}{2}$ where $x = 5$.

By a probability [5] to 1/12600, these two numbers are organized with the same digits in the four defined areas appearance:

Occurrence ranks ⇒	1	2	3	4	5	6	7	8	9	10
$\sqrt{4.5}$ = 2.121320343559642573…3086..	**1**	**2**	**3**	**0**	**4**	**5**	**9**	**6**	**7**	**8**
$((\pi-2)/\pi)^2$ = 0.132045189834…9962…7…	**1**	**3**	**2**	**0**	**4**	**5**	**8**	**9**	**6**	**7**
Occurrence areas ⇒		Area 2		Area 1			Area 4			
	Area 3									

Fig. 18. Constants $\sqrt{4.5}$ and $((\pi\text{-}2)/\pi)^2$: the same figures in the 4 areas of appearance. Probability [5] to 1/12600.

4.4. Other notable constants.

In the constant $\sqrt{4.5}$ the first six digits (0 to 5) of the decimal system are divided into the top six of onset. This phenomenon has a probability to occur than 1/210. However it is observed the same phenomenon in the other five constants, variations of π, described Figures 18 and 19. Also, as in $\sqrt{4.5}$, these numbers have the same configuration property into four arithmetical areas which are by multiples to a divisor of 45. The probability of such an arrangement is to 1/1050 [9] for each number.

Also, with a probability [5] to 1/12600, constants $\sqrt{\pi^2+e^2}$ and $11/\pi^2$ have (as $1/\pi$ and $1/\varphi$) the same distribution of figures inside the four defined arithmetical areas.

* $\sqrt{\pi^2+e^2}$ = 4.154354402313313572948…6…									
1	2	3	4	5	6	7	8	9	10
1	5	4	3	0	2	7	9	8	6
	9 (3 × 3)		**3** (1 × 3)						
		3 (1 × 3)					**30** (10 x 3)		
		15 (5 x 3)							
$11/\pi^2$ = 1.11453302006571548588267409…									
1	2	3	4	5	6	7	8	9	10
1	4	5	3	0	2	6	7	8	9
	9 (3 × 3)		**3** (1 × 3)						
		3 (1 × 3)					**30** (10 x 3)		
		15 (5 x 3)							
$2\left(\sqrt{\pi}/\varphi^2\right)$ = 1.354034255110537068549…									
1	2	3	4	5	6	7	8	9	10
3	5	4	0	2	1	7	6	8	9
	9 (3 × 3)		**0** (0 × 3)						
		6 (2 × 3)					**30** (10 x 3)		
		15 (5 x 3)							

Fig. 19. Constants with the first six digits (0 to 5) of the decimal system in the group of top six. Same arithmetic zones of 1, 2, 3 and 4 digits which are by multiples to 3.

$1/(\pi+1)$ = 0.241453007005223854655569…									
1	2	3	4	5	6	7	8	9	10
2	**4**	**1**	**5**	**3**	**0**	**7**	**8**	**6**	**9**
	5 (1 × 5)		**5** (1 × 5)						
		5 (1 × 5)					**30** (6 x 5)		
		15 (3 x 5)							
$(1/\pi)^3$ = 0.0322515344331994891…6…7…									
1	2	3	4	5	6	7	8	9	10
0	**3**	**2**	**5**	**1**	**4**	**9**	**8**	**6**	**7**
	5 (1 × 5)		**5** (1 × 5)						
		5 (1 × 5)					**30** (6 x 5)		
		15 (3 x 5)							

Fig. 20. Constants with the first six digits (0 to 5) of the decimal system in the group of top six. Same arithmetic zones of 1, 2, 3 and 4 digits which are by multiples to 5.

Together constants $\sqrt{4.5}$ and $((\pi-2)/\pi)^2$, these five other constants, variants derived from π, φ and e, therefore have the same first six and last four digits. The number 0.0123456789101112... , which is the concatenation of the sequence of integers, has obviously his first six digits identical to those numbers. In these, the appearance of ten digits of decimal system are organized also themselves into four arithmetical areas previously defined:

concatenation of the integers sequence = 0.0123456789101112…									
1	2	3	4	5	6	7	8	9	10
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
	3 (1 × 3)		**3** (1 × 3)						
		9 (3 × 3)					**30** (10 x 3)		
		15 (5 x 3)							

Fig. 21. Concatenation of the integers sequence: organization into four arithmetical areas.

This is surely no accident and must be connected with all phenomena introduced in this article. Thus, the number 0.01235711131719…, concatenation of the sequence of prime numbers, with more numbers 0 and 1, also organized themselves in four same arithmetical areas of a multiple to divisor of 45 (also 3):

concatenation of the prime numbers sequence with 0 and 1
= 0.**012357**1113171**9**23293137**4**143475359**6**167717379**8**…

1	2	3	4	5	6	7	8	9	10
0	**1**	**2**	**3**	**5**	**7**	**9**	**4**	**6**	**8**
	3 (1 × 3)		**3** (1 × 3)						
12 (4 × 3)						**27** (9 × 3)			
18 (6 × 3)									

Fig. 22. Concatenation of the prime numbers sequence + 0 and 1: organization into four arithmetical areas.

* Number $\sqrt{\pi^2+e^2}$ is the hypotenuse of a triangle whose sides are π and e:

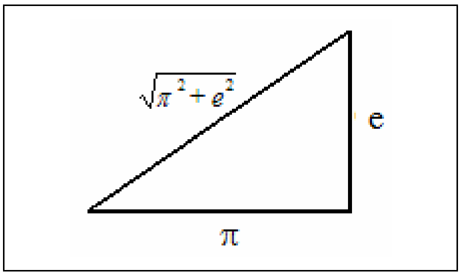


Fig. 23. Triangle whose sides are π and e.

Also, the sine value of this angle (tan = e/π) has remarkable properties:

Sine of the angle whose tangent is $\frac{e}{\pi} = \frac{e}{\sqrt{\pi^2+e^2}}$ = 0.**654321**12**07**366**89**…

1	2	3	4	5	6	7	8	9	10
6	**5**	**4**	**3**	**2**	**1**	**0**	**7**	**8**	**9**
	9 (3 × 3)		**3** (1 × 3)						
9 (3 × 3)						**24** (8 × 3)			
21 (7 × 3)									

Fig. 24. Sine of the angle whose tangent is e/π.

In this sine, the digits apparitions are configured with the same four areas of a multiple to divisor of 45 also (here it is 3). The first six and last four digits are the same as in the constant $\sqrt{2}$ (probability [3] to 1/210). One can also note the unusual regular order of digits occurrence: from 6 to 0 and from 7 to 9.

5. Other constants.

5.1. Constants by ratio to 3/2.

Respective decimal complementarities of π, 1/π, φ and 1/φ are: 4 – π, 1 – (1/π), 2 – φ and 1 – (1/φ). It is (arithmetically) usual in these additional numbers, that digits look into the same configurations described above by four areas which are by multiples to 9 in a ratio to 3/2. However, it is quite strange that the variations of these numbers presented as Figure 25 all have a ratio to 3/2 in order of first appearance of their digits:

Constants, variations of π and φ	Order of appearance of the 10 digits	Sharing out (6 first and 4 last digits)	
$(4-\pi)\times(1-1/\pi)$	5 8 1 6 7 0 4 3 2 9	5 8 1 6 7 0	4 3 2 9
$(2-\varphi)\times(1-1/\varphi)$ *	1 4 5 8 9 0 3 7 6 2	1 4 5 8 9 0	3 7 6 2
$(4-\pi)^2$	7 3 6 8 1 2 0 4 9 5	7 3 6 8 1 2	0 4 9 5
$(2-\varphi)^2$ *	1 4 5 8 9 0 3 7 6 2	1 4 5 8 9 0	3 7 6 2
$\frac{1}{\sqrt{4-\pi}}$	0 7 9 3 2 6 1 8 5 4	0 7 9 3 2 6	1 8 5 4
$\frac{1}{\sqrt{2-\varphi}}$ **	6 1 8 0 3 9 7 4 2 5	6 1 8 0 3 9	7 4 2 5

Fig. 25. Variations of decimal complementarities of π, $1/\pi$, φ and $1/\varphi$: same ratio to 3/2. *because peculiarity of φ, these two variants are identical. ** by the same peculiarity, this variant is equal to φ.

Variants $1/\sqrt{2-\varphi}$ and $1/\sqrt{4-\pi}$ (identical variants of decimal complementarities of φ and π) divide respectively their six first and last four digits as in decimals of $1/\pi$ and of $\sqrt{5}$ (constant whose Phi is derived): probability [3] to 1/210. These two combinations of six and four digits (see below in 5.3) are distinguished by their propensity to appearances in all phenomena presented in this article. Thus, two other formulas, trigonometric configurations and identical variants of Pi and Phi, introduce a remarkable phenomenon. By a ratio to 3/2, digits occurrences of the square of the sine of the angle whose tangent equals to π and those of the square of the sine of the angle whose tangent equals to φ respectively also fall with the same first six and last four digits that decimals of $1/\pi$ and of $\sqrt{5}$ (constant whose Phi is derived): probability [3] to 1/210.

Constants [8]	Order of appearance of the 10 digits	Sharing out (6 first and 4 last digits)	
$\frac{\pi^2}{\pi^2+1} \Rightarrow$ $\sin^2$ of the angle whose tang = π	9 0 8 3 1 6 4 2 7 5	9 0 8 3 1 6	4 2 7 5
$\frac{\varphi^2}{\varphi^2+1} \Rightarrow$ $\sin^2$ of the angle whose tang = φ	7 2 3 6 0 9 4 8 1 5	7 2 3 6 0 9	4 8 1 5

Fig. 26. Remarkable variants of Pi and Phi : same first six and last four figures that decimals of $1/\pi$ and of $\sqrt{5}$

5.2. Constants by four areas which are in multiples to 9.

By a prime ratio (6 and 4 classed digits) to 3/2, in constants, which are variants of π, φ and e, introduced Figures 27, occurrence order of digits organises into the same four arithmetical areas which are by multiples to 9 as those of π and φ (probability [4] to 1/420).

The two first variants in Figure 27 have same first six and last four digits that decimals of $1/\pi$ and of $1/\varphi$: probability [3] to 1/210. Third presented variant has same distribution of six and four digits as constants $\sqrt{5}$, ζ (5), etc. (probability [3] to 1/210 also). These two distributions of figures are unusually more frequent in the constants presented here:

0 1 3 6 8 9 / 2 4 5 7 ($1/\pi$, $1/\varphi$, etc.)
and
0 2 3 6 7 9 / 1 4 5 8 ($\sqrt{5}$, ζ (5), etc.)

Constants [10]	Order of appearance of the 10 digits	Sharing out: areas by 1, 2 and 3 digits	Sharing out: area by 4 digits
$\left(\pi\sqrt{3}\right)^4$	6 8 1 9 3 0 2 5 7 4	6 $\boxed{8\,1}\boxed{9}$ 3 0	2 5 7 4
$\sqrt{4^2+\pi^2}/\pi$ ⇒ 1/cos of angle* which tang = 4/π	6 1 8 9 3 0 2 4 7 5	6 $\boxed{1\,8}\boxed{9}$ 3 0	2 4 7 5
$\sqrt{e^2+\pi^2}/\pi$ ⇒ 1/cos of angle** which tang = e/π	3 2 7 0 6 9 4 8 5 1	3 $\boxed{2\,7}\boxed{0}$ 6 9	4 8 5 1
1/(4φ)	1 5 4 0 8 9 7 3 2 6	1 $\boxed{5\,4}\boxed{0}$ 8 9	7 3 2 6
Log7	8 4 5 0 9 1 2 6 3 7	8 $\boxed{4\,5}\boxed{0}$ 9 1	2 6 3 7
(3φ)/2	4 2 7 0 5 9 8 3 1 6	4 $\boxed{2\,7}\boxed{0}$ 5 9	8 3 1 6
$4/\sqrt{4^2+e^2}$ ⇒ sin of angle which tang = 4/e	8 2 7 0 9 1 6 3 5 4	8 $\boxed{2\,7}\boxed{0}$ 9 1	6 3 5 4
$2\varphi/\sqrt{(2\varphi)^2+5}$ ⇒ sin of angle which *** tang = $2\varphi/\sqrt{5}$	8 2 7 0 1 9 3 5 4 6	8 $\boxed{2\,7}\boxed{0}$ 1 9	3 5 4 6

Fig. 27. Other constants, variations of π and φ and e. * Angle gives squaring the circle.**See 5.3.*** or $\left(\sqrt{5}+1\right)/\sqrt{5}$

In Figure 27, the first two constants have in common to have the same distributions of digits occurrence in their four defined arithmetical areas. This distribution is identical to the value 1 – (1/π) which is the decimal complement of 1/π. With a probability of respectively occurrence [5] to 1/12600, these three numbers are organized with the same digits in the four defined appearance areas. Of course, decimal complementarity of Phi has the same property (see 3.2):

Occurrence ranks ⇒	1	2	3	4	5	6	7	8	9	10
$(\pi\sqrt{3})^4$ = 876.**681**81**930**..**2**..**5**12**7**96…**4**…	**6**	**8**	**1**	**9**	**3**	**0**	**2**	**5**	**7**	**4**
$\sqrt{4^2+\pi^2}/\pi$ = 1.**6189**9**3**..**0**6**2**3…**4**0**7**6**5**… (1/cos of angle whose tang = 4/π)	**6**	**1**	**8**	**9**	**3**	**0**	**2**	**4**	**7**	**5**
1 – (1/π) = 0.**681**6**90**11**3**..**2**09**4**6**7**32**5**…	**6**	**8**	**1**	**9**	**0**	**3**	**2**	**4**	**7**	**5**
Occurrence areas ⇒	Area 3	Area 3: Area 2	Area 3: Area 2	Area 3: Area 1	Area 3	Area 3	Area 4	Area 4	Area 4	Area 4

Fig. 28. Constants $(\pi\sqrt{3})^4$, $\sqrt{4^2+\pi^2}/\pi$ et 1 – (1/π) : same digits in the four occurrence areas. Probability [5] to 1/12600

The values 1/(4φ) and log7 distribute identically also their 10 digits in these same four occurrence areas (see Figure 27).

Also, always with the same very low probability [5] to 1/12600, the last two trigonometric values of Figure 27 have the same common feature:

Occurrence ranks ⇒	1	2	3	4	5	6	7	8	9	10
Sine of angle whose tang = 4/e 0.**827091663**…706**1**5584…	**8**	**2**	**7**	**0**	**9**	**1**	**6**	**3**	**5**	**4**
Sine of angle whose tang = $2\varphi/\sqrt{5}$ 0.**822701898389593218034076**…	**8**	**2**	**7**	**0**	**1**	**9**	**3**	**5**	**4**	**6**
Occurrence areas ⇒	Area 3	Area 2	Area 2	Area 1	Area 3	Area 3	Area 4	Area 4	Area 4	Area 4

Fig. 29. Sine of angles whose tangents are $4/e$ and $2\varphi/\sqrt{5}$: same digits in the four occurrence areas. Probability [5] to 1/12600

5.3. Two preferred combinations.

In connection with the phenomena presented in 4.5.1, both trigonometric configurations of Figure 30 (* and ** in Figure 27), variants of 1/π, are a common phenomenon also. The digits occurrences of the inverse cosine of the angle whose tangent is equal to 4/π and those of the inverse cosine of the angle whose tangent is equal to e/π fall respectively with the same first six and four last digits as in decimals of 1/π and of $\sqrt{5}$ (constant used to form φ): probability to 1/210.

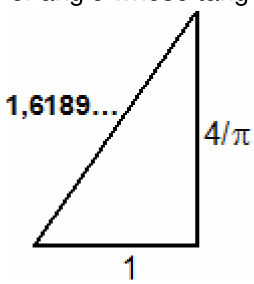
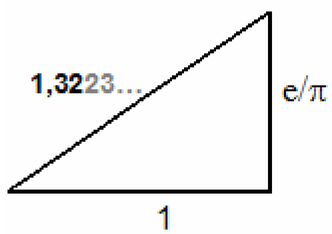
<table>
<tr><td>1/cos of angle whose tang = 4/π</td><td>1/cos of angle whose tang = e/π</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>1.6189…9318660623286240765…</td><td>1.3223…7207696748056509441…</td></tr>
<tr><td>6 | 18 | 9 | 30 | 2475</td><td>3 | 27 | 0 | 69 | 4815</td></tr>
</table>

Fig. 30. Same first six and last four digits as into 1/π et $\sqrt{5}$. Also : same arrangement into 4 areas which are by multiples to 9.

The likelihood of a combination of six and four digits is therefore only to 1/210, so that 99.52% of the combinations of figures occurrence are not the same configuration (first 6 and last 4 figures). However it appears in the phenomena presented in this article, only two combinations of appearances of digits in the constants are much more frequent than is possible by these arithmetical probabilities. These two combinations of six and four digits are (digits ranked in ascending order):

First six digits	Last four digits	constants
0 2 3 6 7 9	**1 4 5 8**	$\varphi^2/(\varphi^2+1)$, $\sqrt{5}$, ζ (5), etc.
0 1 3 6 8 9	**2 4 5 7**	$\pi^2/(\pi^2+1)$, 1/π, 1/φ, etc.

Fig. 31. Two preferred combinations.

There exists a singular relationship between Pi and Phi for the emergence of these two preferred combinations. Indeed, two pairs of two identical formulas using respectively π and φ have their digits occurrence which included in these two combinations (formulas presented above in 5.1):

$\frac{\pi^2}{\pi^2+1}$ $\frac{1}{\sqrt{2-\varphi}}$	$\frac{\varphi^2}{\varphi^2+1}$ $\frac{1}{\sqrt{4-\pi}}$
First six digits and last four digits **0 1 3 6 8 9** **2 4 5 7**	First six digits and last four digits **0 2 3 6 7 9** **1 4 5 8**

Fig. 32. Two pairs of identical formulas using π and φ respectively.

These two combinations of six and four figures occur therefore into variants of Pi and of Phi but without automatic respectivity for Pi or for Phi. Indeed, as described in Figure 32, these two combinations are interchangeable in relation to Pi and Phi. For example, three other formulas in connection with trigonometry and linked either to Pi or to $\sqrt{5}$ produce numbers with one or other of the two preferred combinations of digits occurrence:

Formulas	Numbers	First six digits	Last four digits
Ratio on 360° of angle whose tang = $\sqrt{(4/\pi)^2+1}$ * (1,6189…9318660623…40765…)	0.161938080804195320 57…	1 6 9 3 8 0	4 5 2 7
Ratio on 360° of angle whose tang = $\sqrt{5}$	0.18306988179969249 7…915…	1 8 3 0 6 9	7 2 4 5
tangent of angle which = $360° \times \pi$	1.2337236…23009187…05024	2 3 7 6 0 9	1 8 5 4

Fig. 33. Trigonometric formulas linked to π and to $\sqrt{5}$. * hypotenuse of the angle whose tang = $4/\pi$ (see fig.30).

Each of these two combinations has a probability of occurrence to 1/210, however many constants presented here and not all related to Pi or to Phi part of one or other of these basic combinations (023679/1458 and 013689/2457). Also, many are by arrangement in four arithmetical areas of multiples to 9 and a prime ratio (six and four classified digits) to 3/2. Among 3 628 800 possible combinations, only 1 152 combine these criteria for the one or the other basic combination. This is a probability to 1/3150. Figure 34 lists the constants presented in this paper and who possess these properties.

Constants		Occurrence order of 10 digits	Occurrence areas by 1, 2 and 3 digits	Occurrence areas by 4 digits
Constants by combinations **023679/1458**	$\sqrt{5}$	2 3 6 0 7 9 4 8 1 5	2 | 36 | 0 | 79	4 8 1 5
	ζ (5) (Zeta 5 fonction)	0 3 6 9 2 7 5 1 4 8	0 | 36 | 9 | 27	5 1 4 8
	1/cos of angle whose tang = e/π	3 2 7 0 6 9 4 8 5 1	3 | 27 | 0 | 69	4 8 5 1
	9876543210/0123456789	0 7 2 9 6 3 8 4 1 5	0 | 72 | 9 | 63	8 4 1 5
Constants by combinations **013689/2457**	1 /π	3 1 8 0 9 6 7 5 2 4	3 | 18 | 0 | 96	7 5 2 4
	1/φ (or φ)	6 1 8 0 3 9 7 4 2 5	6 | 18 | 0 | 39	7 4 2 5
	1/cos of angle whose tang = 4/π	6 1 8 9 3 0 2 4 7 5	6 | 18 | 9 | 30	2 4 7 5
	$(\pi\sqrt{3})^4$	6 8 1 9 3 0 2 5 7 4	6 | 81 | 9 | 30	2 5 7 4

Fig. 34. Two preferred combinations: 023679/1458 and 013689/2457.

5.4. Attempting to explain the phenomena.

Study of number x, which is the result [15] of the equation*:

$$x^3 - 2x = (2\varphi - 1)^2$$

* which can also be written: $x^3 - 2x - 5 = 0$

It will not show in this article why the phenomena presented. The author does not a arithmetic explanation which is quite clean. However, research tracks can be envisaged. For example, many of these arrangements appear in singular trigonometric and/or geometric configurations. The author tries to close the phenomena also by links either with the configuration of digits occurrence (for example: same first 6 and last 4 digits) or with the nature of constants or also with these two parameters.

Here it is an example of a research approach that gives other peculiar results. Number x, which is the result [15] of the equation $x^3 - 2x = (2\varphi - 1)^2$ (shown above in Figure 7) produces, by a derived formula, a number which has the same organization of first appearance of figures as into Pi. These two numbers have the same digits in the four areas of occurrence. Recall that the probability of such a phenomenon is only to 1/12600 and so 99.99% possible combinations have not this configuration. This number is $4/(x - 1)$:

Occurrence ranks ⇒	1	2	3	4	5	6	7	8	9	10
π = 3.1415926535897932384….9502…	**1**	**4**	**5**	**9**	**2**	**6**	**3**	**8**	**7**	**0**
$\frac{4}{x-1}$ = 3.654464926915…890178..73…	**6**	**5**	**4**	**9**	**2**	**1**	**8**	**0**	**7**	**3**
Occurrence areas ⇒	Area 3	Area 2 / Area 3	Area 2 / Area 3	Area 1 / Area 3	Area 3	Area 3	Area 4	Area 4	Area 4	Area 4

Fig. 35. π, and variant of x ($x \Rightarrow x^3 - 2x = (2\varphi - 1)^2$): same digits in the 4 occurrence areas. Probability [5] to 1/12600.

This result is similar to that presented below in 6.1 where the number 4/(e-1) (recall e = Neper constant) shares the same phenomenon along with the not fortuitous fraction 9876543210/0123456789. Also, the number $\sqrt{x}/(x-1)$ shares the same phenomenon along with number $\sqrt{e^2+\pi^2}/\pi$ (the inverse cosine of the angle whose tangent is e/π). Both numbers were in fact the same digits in their four respective occurrence areas and also their first six and last four digits are those (like the number 4/(e-1)) of one of the preferred combinations described in the previous chapter:

Occurrence ranks ⇒	1	2	3	4	5	6	7	8	9	10
$\sqrt{x}/(x-1)$ = 1.322..70629382..1334..5…	**3**	**2**	**7**	**0**	**6**	**9**	**8**	**1**	**4**	**5**
$\sqrt{e^2+\pi^2}/\pi$ = 1.32..720769..4805..1..	**3**	**2**	**7**	**0**	**6**	**9**	**4**	**8**	**5**	**1**
Occurrence areas ⇒	Area 3	Area 2 / Area 3	Area 2 / Area 3	Area 1 / Area 3	Area 3	Area 3	Area 4	Area 4	Area 4	Area 4

Fig. 36. Variant of x ($x \Rightarrow x^3 - 2x = (2\varphi - 1)^2$) and variant of π: same digits in the 4 occurrence areas. Probability [5] to 1/12600.

Also, this number x, which is the result [15] of the equation $x^3 - 2x = (2\varphi - 1)^2$, generates other strange phenomena. It may be noted that both values in Figure 37 (shown above in Figure 6) the same first six and last four digits appear:

Constants	First six digits	last four digits
$4\sqrt{\pi}$ = 7.089815403 62…7…	0 8 9 1 5 4	3 6 2 7
$x \Rightarrow [x^3 - 2x - (2\varphi - 1)^2 = 0]$ = 2.09455148154 2326…7…	0 9 4 5 1 8	2 3 6 7

Fig. 37. Constants by ratio to 3/2 with same first 6 and last 4 digits.

Value $4\sqrt{\pi}$ is a geometric value : the perimeter of the square with surface which is equal to Pi. The second value is algebraic [15] and is the result of equation $x^3 - 2x - (2\varphi - 1)^2 = 0$ (or $x^3 - 2x - 5 = 0$). Substituting, in this equation, x by $4\sqrt{\pi}$ and then to a second value x par $4^{\sqrt{1/\pi}}$ there is obtained two other numbers, too, with in order of appearance of the digits of their decimal, a ratio to 3/2:

Constants	First six digits	last four digits
$\left(4\sqrt{\pi}\right)^3-2\left(4\sqrt{\pi}\right)-5$ * = 337,**193**3**60**9**8**998**5**1**7**38798**4**…**2**…	1 9 3 6 0 8	5 7 4 2
$\left(4\sqrt{1/\pi}\right)^3-2\left(4\sqrt{1/\pi}\right)-5$ * = 1,**980**0**5**9**14762**860…611**3**…	9 8 0 5 1 4	7 6 2 3

Fig. 38. Constants which are variants of π with ratio to 3/2.* 5 = $(2\varphi - 1)^2$

Also, the respective order of occurrence for these two new numbers is not random. The first value is organized with the same first six and last four digits as into constants 1/π and 1/φ. These figures are organized into four areas defined above by multiples of a divisor of 45 (here 3). For the second value, the first six and last four digits are identical to the two original values (Figure 37). Also, there are those first six and last four digits in the numbers (described above) $(2 - \varphi)^2$ and 1/4φ. The author does not explain these phenomena but believes they cannot be timely and that this is a research way.

5.5. Other constants by four areas which are multiples to divisor of 45.

5.5.1. Variants of Phi.

In variations of Phi, whose three geometric values, the digits occurrence also organized into four arithmetical areas (defined above) which are by a multiple to divisor of 45:

Perimeter of the square whose surface is equal to φ = $4\sqrt{\varphi}$ = 5,**08**80**7**8**59**805**62**758**4**…**1**…**3**…									
1	2	3	4	5	6	7	8	9	10
0	**8**	**7**	**5**	**9**	**6**	**2**	**4**	**1**	**3**
	15 (3 × 5)		**5** (1 × 5)						
		15 (3 × 5)					**10 (2 × 5)**		
		35 (7 × 5)							

sine of angle whose tangent is 4/φ = $1/\sqrt{(\varphi/4)^2+1}$ = 0,**9270**2**84**9**1**22**3**9**6**… **5**…									
1	2	3	4	5	6	7	8	9	10
9	**2**	**7**	**0**	**8**	**4**	**1**	**3**	**6**	**5**
	9 (3 × 3)		**0** (0 × 3)						
		21 (7 × 3)					**15 (5 × 3)**		
		30 (10 × 3)							

cosine of angle whose tangent is φ = $1/\varphi\sqrt{3-\varphi}$ = 0,**52**5**731**…1**9**133**60**6…90**84**…									
1	2	3	4	5	6	7	8	9	10
5	**2**	**7**	**3**	**1**	**9**	**6**	**0**	**8**	**4**
	9 (3 × 5)		**3** (1 × 3)						
		15 (5 × 3)					**18 (6 × 3)**		
		27 (9 × 3)							

$3\varphi\sqrt{3-\varphi}$ = 5,**7063**3**9**0977709**214**3269**8**6360 …**5** …									
1	2	3	4	5	6	7	8	9	10
7	**0**	**6**	**3**	**9**	**2**	**1**	**4**	**8**	**5**
	6 (2 × 3)		**3** (3 × 3)						
		18 (6 × 3)					**18 (6 × 3)**		
		27 (9 × 3)							

$\varphi/\sqrt{4-\varphi}$ = 1,**0483**8**27**347**5**03770**6**5**9**…**1**…									
1	2	3	4	5	6	7	8	9	10
0	**4**	**8**	**3**	**2**	**7**	**5**	**6**	**9**	**1**
	9 (4 × 3)		**3** (1 × 3)						
		9 (3 × 3)					**18 (6 × 3)**		
		27 (9 × 3)							

Fig. 39. Geometric variants and other variants of φ into 4 areas by multiples to a divisor of 45.

5.5.2. Other constants.

Other mathematical constants also organize the first appearance of their digits into the same four areas (described above) by a multiple to a divisor of 45:

The second order Landau-Ramanujan constant: 0.**58194**8**6**59**3**1**72**9**0**…	5	81	9	46	3720
fractal dimension of the Cantor set (log2/log3): 0.**63092**9**75**357**14**…**8**…	6	30	9	27	5148
fractional area of a Reuleaux triangle: 0.**987**7**0**0**3**9073**6**0**5**3**4**60**1**31**2**…	9	87	0	36	5412
The Khintchine harmonic mean: 1.**745**4**0**5**6**6**2**407**3**46**8**634**9**45…**1**…	7	45	0	62	3891
Lemniscate constant: 5.**24**4**1**1**5**1**08**5842**396**209296**7**…	2	41	5	08	3967

Fig. 40. Constants into 4 areas by multiples to a divisor of 45.

Also more, the fractal dimension of the Cantor set (log2/log3) and two variants of it, a variant of Pi and the fraction 631764/13467:

log2/log3 $\Rightarrow$ 0.**63092**9**75**357**14**…**8**…	6	30	9	27	5148
$(\log 3/\log 2)^{\log 2/\log 3}$ $\Rightarrow$ 1.**3**3**7204819**012280447**65**…	3	72	0	48	1965
$(\log 3/\log 2)^{\log 4/\log 3}$ $\Rightarrow$ 1.**78**8**1**1**6**7**2**7**9**8966**5**7**0**…**43**…	7	81	6	29	5043
$1/(\pi^2-\pi)$ $\Rightarrow$ 0.**148632**32**07**4046**9**18844**5**…	1	48	6	32	0795
631764/13467 $\Rightarrow$ 46.**9120**0**7**12**85364**…	9	12	0	78	5364

Fig. 41. Other constants into 4 areas by multiples to a divisor of 45.

631,764 is a Kaprekar number and the number 13,467 is the number obtained by classing the digits which compose it (digits taken once). Also, the number 631764/13467 is organized with a ratio to 3/2 (27/18), as the number 1467/6174, another fraction incorporating a number of Kaprekar described above in Figure 6.

Also, remarkable variations of the number 8 (2^3) still organized into the same four areas:

$2^3/3^{3/2}$ $\Rightarrow$ 1.**53960**0**71**7**8**3900**2**038691063**4**…	5	39	6	07	1824
$(2^3/3)^{3/2}$ $\Rightarrow$ 4.**3546**4**8**43**1**614538841**2**3**9**61…**0**5**7…**	3	54	6	81	2907
$3\pi/2^3$ $\Rightarrow$ 1.**17809**7**245**09**6**17246442**3**…	1	78	0	92	4563
$2^3/3\pi$ $\Rightarrow$ 0.**84**88**263**63**15**6**7**751**2**41**0**…	8	42	6	31	5720
$3/2^3\pi$ $\Rightarrow$ 0.**1**1**936**6**207**31**8**921**5**…**4**…	1	93	6	20	7854
$\left(\sqrt{2^3}-1\right)^2$ $\Rightarrow$ 3.**34**3**1**4**57**5**0**507**6**1**98**04793**2**…	3	41	5	70	6982

Fig .42. Remarkable variations of the number 8 (2^3) into 4 areas by multiples to a divisor of 45.

6. Variants of e (the Neper constant).

6.1. Variants of e.

In two variants of e described above, $\sqrt{e^2+\pi^2}/\pi$ (reverse sine of angle whose tangent is e/π) and $4/\sqrt{4^2+e^2}$ (sine of angle whose tangent is 4/e), first digits occurrence organized into four areas by multiples of 9 and in a ratio to 3/2.

The first variant has the same first six and last four digits as one of the preferred combinations described in Chapter 5. Variant 4/(e – 1) has exactly the same properties:

$\frac{4}{e-1}$ = 2.**3279068**27**4**7730**5**6975400008**1**…									
1	2	3	4	5	6	7	8	9	10
3	**2**	**7**	**9**	**0**	**6**	**8**	**4**	**5**	**1**
	9 (1 × 9)		**9** (1 × 9)						
	9 (1 × 9)					**18 (2 × 9)**			
27 (3 × 9)									

Fig. 43. Constant 4/(e – 1), variant of e into 4 areas by multiples to 9 and in preferred combinations.

Also, with a probability to only 1/12600, variant 4/(e – 1) distributes the same digits in the four defined areas as the not fortuitous fraction 9876543210/0123456789 :

Occurrence ranks ⇒	1	2	3	4	5	6	7	8	9	10
$\frac{4}{e-1}$ = 2.**3279068**27**4**7730**5**6975…08**1**…	**3**	**2**	**7**	**9**	**0**	**6**	**8**	**4**	**5**	**1**
9876543210/0123456789 = 80.**0**..00**729**0..0**6**6**3**39000…**84**9**1**…**5**…	**0**	**7**	**2**	**9**	**6**	**3**	**8**	**4**	**1**	**5**
Occurrence areas ⇒		Area 2		Area 1			Area 4			
	Area 3									

Fig. 44. Constant 4/(e – 1) and fraction 9876543210/0123456789 : same digits into the 4 occurrence areas.

Also, variants 4/(e + 1) and 1/(e – 1) are organized into four arithmetical areas by multiples to a divisor of 45. The variant 1/(e – 1) is organized into areas by multiples to 9 whose the probability [11] to occur is only to 1/350:

$\frac{4}{e+1}$ = 1.**075**7**6**56**8**5**4**7**9**98048**2**995**3**630…**1**…									
1	2	3	4	5	6	7	8	9	10
0	**7**	**5**	**6**	**8**	**4**	**9**	**2**	**3**	**1**
	12 (4 × 3)		**6** (2 × 3)						
	12 (4 × 3)					**15 (5 × 3)**			
30 (10 × 3)									
$\frac{1}{e-1}$ = 0.**581976**7**0**6869**32**6**4**…									
1	2	3	4	5	6	7	8	9	10
5	**8**	**1**	**9**	**7**	**6**	**0**	**3**	**2**	**4**
	9 (1 × 9)		**9** (1 × 9)						
	18 (2 × 9)					**9 (1 × 9)**			
36 (4 × 9)									

Fig. 45. Constant 4/(e + 1) and 1/(e – 1) : variant of e into 4 areas by multiples to a divisor of 45.

The constant e has thus three variants whose first digits occurrence of their decimals is organized into the four described previously arithmetical areas by multiples to 9 (in ratio to 3/2):

<table>
<tr><th rowspan="2">Constants
Variants of e</th><th rowspan="2">occurrence order of the10 digits</th><th colspan="2">Digits distribution</th></tr>
<tr><th>Areas by 1, 2 and 3 digits</th><th>Areas by 4 digits</th></tr>
<tr><td>$\sqrt{e^2+\pi^2}/\pi$</td><td>3 2 7 0 6 9 4 8 5 1</td><td>3 | 27 | 0 | 69</td><td>4 8 5 1</td></tr>
<tr><td>$4/\sqrt{4^2+e^2}$</td><td>8 2 7 0 9 1 6 3 5 4</td><td>8 | 27 | 0 | 91</td><td>6 3 5 4</td></tr>
<tr><td>$\frac{4}{e-1}$</td><td>3 2 7 9 0 6 8 4 5 1</td><td>3 | 27 | 9 | 06</td><td>8 4 5 1</td></tr>
</table>

Fig. 46. Three numbers, variants of constant e, into 4 areas by multiples to 9 and in ratio to 3/2.

It seems unlikely since the views of many other phenomena in this article that these three arrangements are by chance.

6.2. Variant integrating Pi, Phi, e and i.

It was shown earlier that in many variants of Pi, Phi and e, the first digits appearance of the decimals is organized into two arithmetical preferred areas (see 5.3) in a ratio to 3/2. A formula incorporating these three constants produces a number whose the first digits appearance of the decimals is organized in two arithmetical areas with precisely one of these two combinations of numbers. In this constant, the first six and last four digits are identical to constants $1/\pi$ and $1/\varphi$.

Besides this formula incorporating Pi, Phi, e, but also the imaginary number i, four fundamental mathematical constants, generates a number whose the first digits appearance of the decimals is organised into the same four arithmetical areas by multiples to a divisor of 45 as those described above. This is the formula, variation of a continued fraction of Rogers-Ramanujan:

$$\cfrac{1}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-4\pi}}{1+\ldots}}} = e^{2\pi/5}\left(\sqrt{\varphi+2}+i^2\varphi\right) = 0.998136044598509332149891\ldots7\ldots$$

$$e^{2\pi/5}\left(\sqrt{\varphi+2}+i^2\varphi\right) = 0.998136044598509332149891\ldots7\ldots$$

<table>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>9</td><td>8</td><td>1</td><td>3</td><td>6</td><td>0</td><td>4</td><td>5</td><td>2</td><td>7</td></tr>
<tr><td></td><td colspan="2">9 (3 × 3)</td><td>3 (1 × 3)</td><td></td><td></td><td colspan="4" rowspan="3">18 (6 × 3)</td></tr>
<tr><td colspan="6">15 (5 × 3)</td></tr>
<tr><td colspan="6">27 (9 × 3)</td></tr>
</table>

Fig. 47. Constant integrating Pi, Phi, e and i : same first 6 and last 4 digits as $1/\pi$ and $1/\varphi$. Same four arithmetical areas by multiples to a same divisor of 45 as $1/\pi$ and $1/\varphi$.

For to simplicity the next demonstrations, there will name *r* (*r* as Rogers and Ramanujan) this formula incorporating four fundamental mathematical constants.

Many numbers derived from this formula produce phenomena similar to those described throughout this article. So number $\sqrt{r}/\varphi^2$ has the same arithmetical arrangement as *r* and its first six and last four digits are identical to r (and to $1/\pi$,$1/\varphi$, etc.). Still with a ratio to 3/2 and a arrangement into four arithmetical areas which are previously defined, number $\sqrt[4]{r}/\pi$ described similar arrangements:

$r = e^{2\pi/5}\left(\sqrt{\varphi+2}+i^2\varphi\right) = 0.998136044598509332149891\ldots7\ldots$									
1	2	3	4	5	6	7	8	9	10
9	**8**	**1**	**3**	**6**	**0**	**4**	**5**	**2**	**7**
	9 (3 × 3)		**3** (1 × 3)						
15 (5 × 3)						**18 (6 × 3)**			
27 (9 × 3)									
$\dfrac{\sqrt{r}}{\varphi^2} = 0.381609861405912422 1\ldots7\ldots$									
1	2	3	4	5	6	7	8	9	10
3	**8**	**1**	**6**	**0**	**9**	**4**	**5**	**2**	**7**
	9 (3 × 3)		**6** (2 × 3)						
12 (4 × 3)						**18 (6 × 3)**			
27 (9 × 3)									
$\dfrac{\sqrt[4]{r}}{\pi} = 0.318161453533359863557114298 70\ldots$									
1	2	3	4	5	6	7	8	9	10
3	**1**	**8**	**6**	**4**	**5**	**9**	**7**	**2**	**0**
	9 (3 × 3)		**6** (1 × 3)						
12 (4 x 3)						**18 (6 × 3)**			
27 (9 × 3)									

Fig. 48. r and two variants of e into 4 areas by multiples to a same divisor of 45.

Also, the decimal complementarity of $\sqrt{r}/\varphi^2$ (which is equal to 1 – ($\sqrt{r}/\varphi^2$) and r, distribute their own digits in the four defined areas. Probality [5] to only 1/12600.

The six numbers φ/r, $r\times\varphi$, φ/r^2, r/φ^4, $\varphi/\sqrt{r}$ and $\varphi/\sqrt[3]{r}$, all variants derived to r, have the first digits occurrence of decimals organized into four arithmetical areas by multiples to a divisor of 45:

Constants [13]	occurrence order of the10 digits	Digits distribution: Areas by 1, 2 and 3 digits				Digits distribution: Areas by 4 digits
$\dfrac{\varphi}{r}$	6 2 1 0 5 4 7 9 8 3	6	2 1	0	5 4	7 9 8 3
$r\times\varphi$	6 1 5 0 8 4 7 9 2 3	6	1 5	0	8 4	7 9 2 3
$\dfrac{\varphi}{r^2}$	6 2 4 0 8 7 1 9 3 5	6	2 4	0	8 7	1 9 3 5
$\dfrac{r}{\varphi^4}$	1 4 5 6 2 0 8 3 9 7	1	4 5	6	2 0	8 3 9 7
$\dfrac{\varphi}{\sqrt{r}}$	6 1 9 5 4 0 7 2 3 8	6	1 9	5	4 0	7 2 3 8
$\dfrac{\varphi}{\sqrt[3]{r}}$	6 1 9 0 4 5 2 3 8 7	6	19	0	4 5	2 3 8 7

Fig. 49. Six variants derived to r which organize into four arithmetical areas by multiples to a divisor of 45.

Recall that only one combination of digits occurrence onto eighteen has this property and that 94.44% of possible combinations have not this configuration. Also, the last two numbers presented in Figure 49 are organized, with a probability [3] to 1/210, with the same first six and last four digits.

7. Phi+ : a number which is a cousin of Phi.

A number which is a variant of the Golden Number (Phi) has some remarkable properties which are directly in connection to the phenomena introduced above. The Golden Number is given by the formula:

$$\frac{\sqrt[2]{5}+1}{2}$$

Substituting, in this formula, the square root of 5 by the cube root of 5 there is obtained the number:

$$\frac{\sqrt[3]{5}+1}{2} = 1,\mathbf{354987}9733383484946765544362719\ldots0\ldots$$

This number has the same arithmetical arrangements as Phi: in these, the appearances of the figures are organized in the same four occurrence areas (described above) to form sums whose values are by multiples of 9. The probability [11] that the digits occurrences are organized into this four areas of multiples of 9 is only to 1/350. 99.71% of all possible combinations of the appearance of ten digits of the decimal system have not this arithmetical arrangement. There is very unusual and not fortuitous that Phi [($\sqrt[2]{5}$ +1)/2] and Phi+ * [($\sqrt[3]{5}$ +1)/2] possess these properties simultaneously.

$\varphi_+ = (\sqrt[3]{5}+1)/2$ = 1.**354987**9733383484946765544362719…**0**									
1	2	3	4	5	6	7	8	9	10
3	**5**	**4**	**9**	**8**	**7**	**6**	**2**	**1**	**0**
	9 (1 × 9)		**9** (1 × 9)						
	18 (2 × 9)					**9 (1 × 9)**			
36 (4 × 9)									

Fig. 50. ($\sqrt[3]{5}$ +1)/2 or φ_+ [12].

*This number is provisionally named [12] Phi+ and it is written φ_+. This number creates many other remarkable numbers in many derived forms. Variants of this number presented below have the same singular arrangements as those described above in the article. Many of these variants have unusual connections with Pi and Phi (the Golden Number).

7.1. Formula $2(\varphi_+^2 + \varphi_+)$

So, formula $2(\varphi_+^2 + \varphi_+)$ gives the number :

6.381960762459827011459611425156 7…

This number has the same arrangement of digits occurrence into four areas by multiples of 9 and a ratio to 3/2 as the constants $1/\pi$ and $1/\varphi$ (probability [4] to 1/420). Too, in this distribution, its first six et last four digits are the same as in $1/\pi$ and $1/\varphi$ (probability [3] to 1/210).

$2(\varphi_+^2 + \varphi_+)$ = 6.**3819607**6**245**…									
1	2	3	4	5	6	7	8	9	10
3	**8**	**1**	**9**	**6**	**0**	**7**	**2**	**4**	**5**
	9 (1 × 9)		**9** (1 × 9)						
	9 (2 × 9)					**18 (2 × 9)**			
27 (3 × 9)									

Fig. 51. formula $2(\varphi_+^2 + \varphi_+)$

Still more, with probability [5] to 1/12600, This number organized with the same digits occurrence into the four appearance areas as numbers 1-(1/π), $(\pi\sqrt{3})^4$ and cosine reverse of the angle whose tangent is equal to 4/π (three numbers described above in 5.2).

7.2. Formula $8 - 2(\varphi_+^2 + \varphi_+)$

By subtracting the number $2(\varphi_+^2 + \varphi_+)$ from the superior second whole number (so from 8) there is obtained a number very similar to the Golden Number * (but not the Golden Number):

$$8 - 2(\varphi_+^2 + \varphi_+) = \mathbf{1.6180392375401729885403885748433\ldots}$$

*φ = 1.61803398874989484820458683436564…

This number is organized in its digits appearances, just as the Golden Number:

$8 - 2(\varphi_+^2 + \varphi_+)$ = **1.61803923754**…									
1	2	3	4	5	6	7	8	9	10
6	**1**	**8**	**0**	**3**	**9**	**2**	**7**	**5**	**4**
	9 (1 × 9)		**0** (0 × 9)						
18 (2 × 9)						**18 (2 × 9)**			
27 (3 × 9)									
φ = 1.**61803**3**9**88**74**9894848**2**04**5**868…									
1	2	3	4	5	6	7	8	9	10
6	**1**	**8**	**0**	**3**	**9**	**7**	**4**	**2**	**5**
	9 (1 × 9)		**0** (0 × 9)						
18 (2 × 9)						**18 (2 × 9)**			
27 (3 × 9)									

Fig. 52. Formula $8 - 2(\varphi_+^2 + \varphi_+)$: number extremely similar to the Golden Number.

Thus, a variation of Phi+ [12] produced a number almost identical to Phi, whose appearance of its decimal digits is almost identical to Phi, but that is different from Phi. This reinforces the idea that the organization of digits occurrences in the fundamental constants is not by chance.

7.3. Formula $1 - (1/\sqrt[3]{5})$

The formula $1 - (1/\sqrt[3]{5})$ gives a number whose the organization of digits occurrences is very close to Pi:

$$1 - \frac{1}{\sqrt[3]{5}} = 0{,}\mathbf{415}1\mathbf{96}45\mathbf{23}5\mathbf{7}4267\mathbf{8}68\ldots\mathbf{0}\ldots$$

This number has the same appearance of the first six and last four digits as Pi. Also, it is organized as Pi in four areas by multiples of 3:

$1 - (1/\sqrt[3]{5})$ = 0.**415**1**96**45**23**5**7**4267**8**68…**0**…									
1	2	3	4	5	6	7	8	9	10
4	**1**	**5**	**9**	**6**	**2**	**3**	**7**	**8**	**0**
	6 (2 × 3)		**9** (3 × 3)						
12 (4 × 3)						**18 (6 × 3)**			
27 (9 × 3)									
π = 3.**14**1**5926**5**3**5**8**9**7**932384626433832795**0**2…									
1	2	3	4	5	6	7	8	9	10
1	**4**	**5**	**9**	**2**	**6**	**3**	**8**	**7**	**0**
	9 (3 × 3)		**9** (3 × 3)						
9 (3 × 3)						**18 (6 × 3)**			
27 (9 × 3)									

Fig. 53. Formula $1 - (1/\sqrt[3]{5})$ and π: very similar organization of digits occurrences.

7.4. Formula $1 - (\sqrt[3]{5}/2)$.

The formula ($\sqrt[3]{5}/2$), close formula to Phi+, gives the number:

$$\frac{\sqrt[3]{5}}{2} = 1.854987973338348494676554436271 9\ldots0$$

This number has the same arrangement of digits occurrence into four areas by multiples of 9 as Phi+.

The formula $1 - (\sqrt[3]{5}/2)$, which is the decimal complementarity of the previous, gives a number whose the organization of digits occurrences is very close to Pi:

$1 - (\sqrt[3]{5}/2)$ = 0.145012026…50532…6372807…9…									
1	2	3	4	5	6	7	8	9	10
1	4	5	0	2	6	3	7	8	9
	9 (1 × 9)		0 (0 × 9)						
9 (1 × 9)						27 (3 × 9)			
18 (2 × 9)									
π = 3.14159265358979323846264338327950 2…									
1	2	3	4	5	6	7	8	9	10
1	4	5	9	2	6	3	8	7	0
	9 (1 × 9)		9 (1 × 9)						
9 (1 × 9)						18 (2 × 9)			
27 (3 × 9)									

Fig. 54. Formula $1 - (\sqrt[3]{5}/2)$ and π: very close organization of digits occurrences.

Still more, with probability [5] to 1/12600, This number $1 - (\sqrt[3]{5}/2)$ is organized with exactly the same four digits areas as the number $3/[(4/\pi)^2 +1]$, a variant of Pi whose the arrangement of digits occurrences is very close to Pi also :

Occurrence ranks ⇒	1	2	3	4	5	6	7	8	9	10
$1 - (\sqrt[3]{5}/2)$ = 0,145012..6..32…637 28..9…	1	4	5	0	2	6	3	7	8	9
$3/[(4/\pi)^2 +1]$ = 1.144540625523 49..87…	1	4	5	0	6	2	3	9	8	7
Occurrence areas ⇒		Area 2		Area 1			Area 4			
	Area 3									

Fig. 55. $1 - (\sqrt[3]{5}/2)$ and $3/[(4/\pi)^2 +1]$: same organization of digits occurrences. Probability [5] to 1/12600.

This number $3/[(4/\pi)^2 +1]$ is not fortuitous. A number whose the formula is very close has its digits occurrences organized into a oddly close configuration. This is the number $4/[(4/\pi)^2 +1]$:

$4/[(4/\pi)^2 +1]$ = 1.5260541673646549544959792391 8…									
1	2	3	4	5	6	7	8	9	10
5	2	6	0	4	1	7	3	9	8
			0 (0 × 9)						
18 (2 × 9)						27 (3 × 9)			
18 (2 × 9)									

Fig. 56. $4/[(4/\pi)^2 +1]$: oddly close configuration to $3/[(4/\pi)^2 +1]$ (see Fig. 55).

7.5. Other derived formulas to Phi+.

Formulas $\varphi_+^2 - \varphi_+$ and $1/(\varphi_+^2 - \varphi_+)$ are organized, in the occurrence of its digits, into four areas by multiples to a divisor of 45 (here 3). This is with a probability [6] to 1/18.

$\varphi_+^2 - \varphi_+$ = 0.48100443455321651637669…									
1	2	3	4	5	6	7	8	9	10
4	8	1	0	3	5	2	6	7	9
	9 (3 × 3)		0 (0 × 3)						
		12 (4 × 3)					24 (8 × 3)		
		21 (7 × 3)							
$1/(\varphi_+^2 - \varphi_+)$ = 2.07898291193272516871205528714…									
1	2	3	4	5	6	7	8	9	10
0	7	8	9	2	1	3	5	6	4
	15 (5 × 3)		9 (3 × 3)						
		3 (1 × 3)					18 (3 × 9)		
		27 (3 × 9)							

Fig. 57. formulas $\varphi_+^2 - \varphi_+$ and $1/(\varphi_+^2 - \varphi_+)$.

With the same occurrence probability [6] to 1/18, formulas presented Figure 58, variants of Phi+, are organized in the same configurations into four areas by multiples to a divisor of 45:

Constants	Areas by 1, 2 and 3 digits	Area by 4 digits
$\frac{1}{\varphi_+^5}$ = 0.21893883232376394302636…5…	2 18 9 37	6 4 0 5
$\frac{1}{(\varphi_+ - 1)^5}$ = 177.391089444532774546…	3 91 0 84	5 2 7 6
$\frac{(\varphi_+ - 1)^5}{\varphi_+^5}$ = 0.00123421550095…05825…370…6…	0 12 3 45	9 8 7 6

Fig. 58. Other derived formulas to φ_+.

The last formula $(\varphi_+ - 1)^5/\varphi_+^5$, ratio of the first two, has the same first six and last four digits as the constant $\sqrt{4.5}$ and other numbers presented in 4.4 which, remember this, split their first six digits from 0 to 5 and their last four from 6 to 9. One can also note the unusual regular order of digits occurrence for this formula: 0-1-2-3-4-5 and 9-8-7-6. What makes that this number has the same digits in the four defined arithmetical areas (probability [5] to 1/12600) as the concatenation (presented in 4.4) of the integers sequence (0,01234567891011…).

The formula $\varphi_+^2/(\varphi_+^2 - 1)$, with a probability [3] to 1/210, has the same occurrences of first six and last four digits as numerous constants introduced above in this paper which mainly 1/Pi, Phi, etc. This formula can be closed to the trigonometric formula $\pi^2/(\pi^2 + 1^2)$ presented above in 5.1. For indeed, this formula can be written with the imaginary number i :

$$\frac{\varphi_+^2}{\varphi_+^2 + i^2}$$

Constants	first six digits	last four digits
$\varphi_+^2/(\varphi_+^2 + i^2)$ = 2.19618311190417…2…5…	1 9 6 8 3 0	4 7 2 5
$\pi^2/(\pi^2 + 1^2)$ = 0.908000331649624767544…	9 0 8 3 1 6	4 2 7 5

Fig. 59. Two close formulas with same distribution of the first six and last four digits.

7.6. Phi+ and ‘‘The hard hexagon constant’’.

The number ($\sqrt[3]{5}$ +1)/2 (which is Phi+) and the number $\sqrt[4]{5}$ are organized with the same first six and last four digits. Still, this combination of six and four digits is the same as in ‘‘The hard hexagon constant’’ [14]:

Constants	first six digits	last four digits
Phi+ = ($\sqrt[3]{5}$ +1)/2 = 1.3549879…946765…362719…0	3 5 4 9 8 7	6 2 1 0
$\sqrt[4]{5}$ = 1.495348781221220541911…6…	4 9 5 3 8 7	1 2 0 6
Hard hexagon constant = 1,39548597247930 2…006…1…	3 9 5 4 8 7	2 0 6 1

Fig. 60. Three constants with same combination of the first six and last four digits.

Note: into the square root of "The hard hexagon constant" [14] (1.181306891742913 15…), appear the same first six and last four digits as into the numbers $1/\pi$ and φ (one of two preferred combinations of occurrences described above in 5.3). This is yet in all likelihood not a fortuitous phenomenon.

7.7. Phi+, Phi and e.

Some variants of Phi+ associate to Phi show other strange phenomena including unusual similarities to variants of the constant e (Neper constant):

constant	value	first six digits	last four digits
$\frac{\varphi}{\sqrt{\varphi_+}}$	= 1.39001638632480395…7… ⇒	390168	2457
	Ratio to 3/2 Same first six and last four digits as $1/\pi$, $1/\varphi$, etc.		
$\sqrt{\frac{\varphi}{\sqrt{\varphi_+}}}$	= 1.1789895615843 2…0… ⇒	1 78 9 56	4320
⇓	arrangement into 4 areas by multiples to a divisor of 45 Same first six and last four digits as:		⇓
$\frac{1}{e-1}$	= 0.58197670686932 64… ⇒	5 81 9 76	0324
$\frac{1}{1-\sqrt{\frac{\varphi}{\sqrt{\varphi_+}}}}$	= 1.218011310464856445839…7… ⇒	2 18 0 34	6597
⇓	arrangement into 4 areas by multiples to a divisor of 45 Same first six and last four digits and same digits into the 4 occurrence areas as:		⇓
$1-\frac{1}{e-1}$	= 0.418023293130673 5… ⇒	4 18 0 23	9675
$1-\sqrt{\frac{\varphi}{\sqrt{\varphi_+}}}$	= 0.821010438415672 8…9… ⇒	8 21 0 43	5679
⇓	Same first six and last four digits as its inverse :		⇓
$\frac{1}{1-\sqrt{\frac{\varphi}{\sqrt{\varphi_+}}}}$	= 1,218011310464856445839…7… ⇒	2 18 0 34	6597

Fig. 61. Some variants of Phi+.

8. Other findings.

In order not to overload this article by too of many shows, the author has here presented the findings only which have most significant connections with the phenomena described. Here are just some examples of investigations reinforcing the idea that the first appearance of the ten decimal digits inside remarkable constants is not random.

8.1. Landau-Ramanujan Constant.

The Landau-Ramanujan Constant itself organizes by a ratio [1] to 3/2 (27/18). Three variants of this constant (named C here) have the same property whose constant C^2 which itself organizes besides into four arithmetical areas by multiples of a divisor of 45:

Constants	first six digits	last four digits
Landau-Ramanujan Constant (*C*) = 0,**7642**2**3**6**5**35**89**22**0**66299…**1**…	7 6 4 2 3 5	8 9 0 1
$\frac{3C}{2}$ = 1,**1463**3**5**4**80**383830**9**944860…**2**…**7**…	1 4 6 3 5 8	0 9 2 7
$\frac{1C}{2}$ = 0,**3821**11182**6794**61**0**33149**5**…	3 8 2 1 6 7	9 4 0 5
C^2 = 0,**584037**7**92**7052571**1**443348483**6**…	5 84 0 37	9 2 1 6

Fig. 62. Landau-Ramanujan constant and three variants by a ratio to 3/2 (27/18).

8.2. Number 33 and number Pi.

The order of first appearance of the ten digits of the square root of 33 generates four arithmetical areas previously defined (by multiples of a divisor of 45). In association to Pi, this number gives some others with similar characteristics:

Constants	Occurrence order of 10 digits	Distribution	
		Areas by 1, 2 and 3 digits	Area by 4 digits
$\sqrt{33}$ = 5,**74**4**562**6465**380**2865**9**8506**1**14682…	7 4 5 6 2 3 8 0 9 1	7 45 6 23	8091
$\sqrt{33}/\pi^2$ = 0,58204588685479348660096013725…	5 8 2 0 4 6 79 3 1	5 82 0 46	7931
$\pi^2/\sqrt{33}$ = 1,7180775993516745836069264697…	7 1 8 0 5 9 3 6 4 2	7 18 0 59	3642
$\left(\sqrt[2]{33}/\pi\right)^2$ = 3,343599060197146457648022285…	3 4 5 9 0 6 1 7 8 2	3 45 9 06	1782
$\left(\sqrt[3]{333}/\pi\right)^3$ = 10,739760966255429898412543545…	7 3 9 6 0 2 5 4 8 1	7 39 6 02	5481

Fig. 63. Variants* dérived to $\sqrt{33}$ into four arithmetical areas by multiples of a divisor of 45.

* The last formula in Figure 63 is not a variant to 33 but its arithmetic construction $\left(\sqrt[3]{333}/\pi\right)^3$ is close to formula $\left(\sqrt[2]{33}/\pi\right)^2$. Still, inside this number, the first six and last four digits occurrences are the same as one of the two preferred combinations previously highlighted in 5.3.

8.3. Ratio 1/7.

Many rational numbers are formed by a sequence of repeated decimals, this is one of their characteristics. Very often this repetitive sequence consists to digits whose the sum is a multiple of 9. The first rational number (among inverses of integers) to be formed from such a sequence is the number 1/7 whose the repeated sequence of its decimals is formed by the digits 1-4-2-8-5-7 (1/7 = 0.142857142857…). The addition of these six different figures gives 27 and adding the four missing numbers (0-3-6-9) gives 18. This gives a ratio to 3/2 between these two sets of digits. In ranking order of magnitude the first six digits gives the number 124,578 and the ratio 124578/142857 gives a number whose digits are formed by repetitive series 8-7-2-0-4-6. This sequence is organized into three areas by multiples of 9 which are identical to those described in this paper and the four missing digits form a fourth area by a multiple of 9 and in a prime ratio to 3/2 with this series:

124578/142857 = 0.**872046**872046872046…									
1	2	3	4	5	6	four missing digits			
8	**7**	**2**	**0**	**4**	**6**	**1**	**3**	**5**	**9**
	9 (1 × 9)		**0** (0 × 9)						
18 (2 × 9)						**18 (2 × 9)**			
27 (3 × 9)									

Fig. 64. Rational number 124578/142857 derived to 1/7.

The author find that this phenomenon is not accidental and is linked to all other phenomena introduced in this article. So, this is a possible research way to explain these singular phenomena.

8.4. The Fibonacci series.

By dividing each number in the Fibonacci sequence by 10^n, where n is the rank of each number, then summing these numbers, there is obtained a number that tends toward the rational number 10/89. This number has the same organization of digits occurrence into four areas by multiples to a divisor of 45 also:

Fibonacci series	Fibonacci series by 10^{-n}	Values additions (up to ∝)
1	0. **1**	
1	+ 0. 0**1**	
2	+ 0. 00**2**	
3	+ 0. 000**3**	
5	+ 0. 0000**5**	
8	+ 0. 00000**8**	= 0.**1**1**2359**55**0**5**6**1**7**97752**8**08…**4**… $= \frac{10}{89}$
13	+ 0. 000000**13**	
21	+ 0. 0000000**21**	
34	+ 0. 00000000**34**	
55	+ 0. 000000000**55**	
…	+ 0. 0………………	
	0.112359550561…	

1	2	3	4	5	6	7	8	9	10
1	**2**	**3**	**5**	**9**	**0**	**6**	**7**	**8**	**4**
	5 (1 × 5)		**5** (1 × 5)						
10 (2 × 5)						**25 (5 × 5)**			
20 (4 × 5)									

Fig. 65. Special addition of Fibonacci numbers: 10/89 is organized into four areas by multiples to a divisor of 45.

It is well known that the Fibonacci sequence gives the number Phi, main topic of this article. This further demonstration gives credence to the idea that the order of first appearance of the ten digits forming the decimals of numerous mathematical constants is not fortuitous.

9. Prime numbers, decimal system and 3/2 ratio.

In parallel to the study of the order of the first occurrence of the ten digits in decimals of many mathematical constants and particular numbers described in this paper, remarkable properties about the formation of the ten digits and the scripture of primes numbers are obliged to be introduced here.

9.1. Formation of the ten digits (according to prime numbers).

The ten digits: 0 1 2 3 4 5 6 7 8 9

Six primes or fundamental numbers (0 and 1): 0 1 2 3 5 7. Sum equal to 18
Four not primes and not fundamental numbers : 4 6 8 9. Sum equal to 27

So a ratio to 18/27 (so 2/3).

Four not primes: 4 6 8 9

The four not primes	Combinations (of primes)	So with the primes (one time counted)	Quantities of used primes	
4	2 x 2	2	2	(2 times prime 2)
6	2 x 3	2 + 3	2	(prime 2 and prime 3)
8	2 x 2 x 2	2	3	(3 times prime 2)
9	3 x 3	3	2	(2 times prime 3)
		12	9	

Fig. 66. Formation of the four not primes according to primes.

The six primes or fundamental numbers are (of cause) just combinations of 6 primes (themselves).
The four not primes are combinations of 9 primes (see fig. 66).

So a ratio to 6/9 (so 2/3).

Sum of the primes to form the six primes or fundamental numbers = 18
Sum of the primes (one time enumerated) to form the four not primes = 12

So a ratio to 18/12 (so 3/2).

	Primes (and fundamental numbers)	Not primes	ratio
Numbers	0 1 2 3 5 7	4 6 8 9	
Sum of the 6 primes Sum of the 4 not primes	18	27	18/27 so 2/3
Sum of primes to form the 6 primes Sum of primes to form the 4 not primes	18	12	18/12 so 3/2
Combinations of n primes	6	9	6/9 so 2/3

Fig. 67. 3/2 ratio3/2 in the formation of the ten digits (according to prime numbers).

9.2. Digit scripture of the primes

All prime numbers have only digits 1/2/3/5/7/9 in latest position, so 6 digits are possible.
All prime numbers have not digits 0/4/6/8 in latest position, so 4 digits are not possible.

Sum of 1 – 2 – 3 – 5 – 7 – 9 is equal to 27 and sum of 0 – 4 – 6 – 8 is equal to 18.

So there are a twice ratio 3/2 about digit scripture of the primes : 6 and 4 digits possible or not possible in latest position and 27 and 18 the sums of these 6 and 4 digits.

10. Conclusion.

The order of the first occurrence of the ten digits forming the decimals of many mathematical constants is not random. Into the constants which are introduced in this paper, always identical areas of one, two, three and four digits have sums which are by multiples to a same divisor of 45 (according to the constants: 3, 5 or 9). This occurrence areas are always : in the occurrence rank 4 for the one digit area, in the occurrence ranks 2 and 3 for the two digits area, in the occurrences ranks 1, 5 and 6 for the three digits area and in the occurrence ranks 7, 8, 9 and 10 for the four digits area:

occurrences ranks ⇒	1	2	3	4	5	6	7	8	9	10
occurrences areas ⇒		Area 2		Area 1			Area 4			
	Area 3									

Fig. 68. Identification of four occurrence areas of ten digits of the decimal system in constants.

The occurrence probability of this basic configuration is only to 1/18 and 94.44 % of possible configurations have not this arrangement. However, the constants Pi, 1/Pi, Phi (and 1/Phi), numbers $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$, number $\sqrt{4.5}$ (square root of the average of ten digits of the decimal system), The Zeta 5 function and very many variants of these numbers here introduced whose Phi+ [12] and some variants of the Neper constant (e) are organized into this basic configuration. A large proportion of these numbers are values related to the geometry field.

Also, a high proportion (higher probabilities) of these numbers, including the major Neper constant (e) has a ratio to 3/2 in the digits appearance of their decimals (six first against four occurred digits).

The number Pi and the Golden Number (Phi) possess these properties and they have particularity to reproduce these arithmetical faculties for their inverses. The inverse to number Pi and the inverse to Golden Number are closed by a more still singular phenomena because, for these two fundamental constants of Mathematics, by a probability to only 1/12600, the same figures occurs into the four defined digits occurrence areas of their decimals.

Also, the observation that these singular phenomena are verified for many other constants, whose the numbers $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ (square roots of the first three prime numbers) and for variants of the Neper constant (e) confirms that the order of first appearance of digits in the decimal of constants which are presented in this paper is not random.

In conclusion, the author proposes to consider the existence of a new family of numbers having the characteristics described in this article. Family of numbers which the number Pi and the Golden Mean are the most significant representatives. Also, the author recalls and insists that this new field of research investigates, in numbers, only the first appearance of the ten digits of the decimal system and suggests that it is not fruitful to extend the investigations to the following appearances. The fact, not presented here but experienced by the author, that these investigations are sterile paradoxically reinforces the idea that the phenomena introduced in this study must be subject to greater attention.

Since the publication of this article, the author continues his researches on these intriguing arithmetic phenomena about the first occurrences of the ten digits of the decimal system in other significant numbers and constants. These new investigations are presented here: **new discoveries**

It is in an unstructured form. These new investigations will be gradually integrated into a future version of the article. The author invites anyone to participate in these new researches. If you think you've discovered some interesting thing in link to this, you can introduce your own discoveries to the author. They will then be presented (with your references) on this site.

Annexe

[1] There are 3,628,800 different combinations in the distribution of digits occurrences in decimals of constants. 311,040 combinations have a ratio to 3/2 (27/18). This is only to 1/11.66 and therefore 91,43 % of the possible combinations are not this ratio.

[2] The probability that the constant π and $1/\pi$ have simultaneously a ratio to 3/2 (see [1]) is to 1/23.66.

[3] Among the 3,628,800 different combinations, 17,280 have the same distribution of 6 and 4 digits, this is only to 1/210 and therefore 99.52 % of digits occurrence combinations have not the same configuration (of 6 and 4 digits).

[4] Among the 3,628,800 different combinations, 8,640 have the same arithmetical configuration into 4 areas by multiples of 9 and a ratio to 3/2. This is only to 1/420 and 99.76 % of possible combinations have not this configuration.

[5] Among the 3,628,800 combinations, only 288 have the same digits distributed into the 4 defined arithmetical areas. This is only to 1/12600 and **99.99 % of possible combinations have not this configuration**.

[6] Among the 3,628,800 combinations, 201,600 have the same 4 areas of digits whose the sums are by multiples of the same numbers (3, 5 or 9 in according to combinations). This is only to 1/18 and 94.44 % of possible combinations have not this configuration.

[7] Possible combinations = 9 x 7 x 5 x 3 x 1 = 945.

[8] - $\sin^2$ of angle whose tangent = π : 0.**908**000**3**3**164**96**2**4**7**67**5**44… $\pi^2/(\pi^2 + 1)$
- $\sin^2$ of angle whose tangent = φ : 0.**72360**67**9**77**4**997**8**96964 09**1**…**5**… $\varphi^2/(\varphi^2 + 1)$

[9] Among the 3,628,800 combinations, 3,456 have in same time the first 6 digits of decimal system (from 0 to 5) in the first six ranks of occurrence and the same areas of four digits whose the sums are by multiples of the same divisor of 45 (3 or 5 in according to combinations). This is to 1/1050.

[10] - $(\pi\sqrt{3})^4$: 876.**681**81**930**60**2**193**5**12**7**96299**4**198…
- 1/cos of angle whose tangent is $4/\pi$: 1.**6189**9**3**1866**0**6**2**32862**4**0**7**6**5**967…
- 1/cos of angle whose tangent is e/π : 1.**32**23**7**2**0**7**69**67**48**0**5**650944**1**395…
- $1/4\varphi$: 0.**154**5**08**4**97**18747**3**71**2**05114**6**708…
- $3\varphi/2$: 2.**42705**0**9831**24842272 30**6**880…
- sin of angle whose tangent is $4/e$: 0.**827091**6**63**…70615**5**58**4**…
- sin of angle whose tangent is $2\varphi/\sqrt{5}$: 0.**82**2**701**8**9**8**3**89**5**93218 03**4**07**6**…

[11] Among the 3,628,800 combinations, 10,368 have the same arithmetical configuration into 4 areas by multiples of 9 (with or without ratio to 3/2). This is to 12/350 and 99.71 % of possible combinations have not this configuration.

[12] Pending to a more formal name, the author proposes to temporarily call the number $\frac{\sqrt[3]{5}+1}{2}$ (1.3549879733383484946765544362719…0…), variation of the Golden Number, Phi + and to represent this one by the symbol φ_+.

[13] $r = e^{2\pi/5}\left(\sqrt{\varphi+2}+i^2\varphi\right)$: For simplicity the demonstrations, this formula is named r as *Rogers* and *Ramanujan*.

φ/r = 1.**62105**556**4**0245**7**4**9**55**83**87576785698

$r\times\varphi$ = 1.**6150**1**8**0**4**5556**7**68**9912**054**3**19315883

φ/r^2 = 1.**62408**2**7**8**1**8**9**8**3**623986718091080939…**5**…

r/φ^4 = 0.**14562**6**08**6**3**2223**9**68**7**13316326201939

$\varphi/\sqrt{r}$ = 1.**61954**4**07**17**2**015**3**4561999262513712…**8**…

$\varphi/\sqrt[3]{r}$ = 1.**61904**0**5**54**2**02**3**005**8**292**7**3871877919

[14] ''The hard hexagon constant'' : the informed reader must know that constant whose author (self educated researcher) found no clear definition.

[15] x is the result of the equation $x^3 - 2x = (2\varphi - 1)^2$ which is equal to the function $x^3 - 2x - 5 = 0$ which is used in the Newton method.